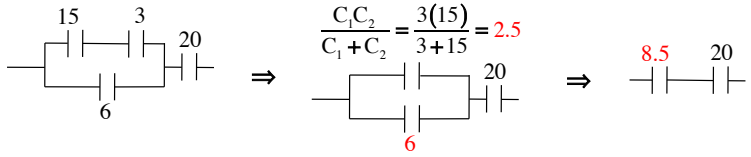
Problem 26.23

Note that the equivalent cap for two series capacitors can be written as:

$$\left(\frac{1}{C_{equ}}\right)^{-1}=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}=\left(\frac{C_2}{C_1C_2}+\frac{C_1}{C_1C_2}\right)^{-1}=\left(\frac{C_1+C_2}{C_1C_2}\right)^{-1}$$

$$\Rightarrow\quad C_{equ}=\frac{C_1C_2}{C_1+C_2}$$

a.) The equivalent capacitance (I think that doing a problem like this algebraically first is the best way to go, but for the sake of brevity, I'll do it solely with numbers):

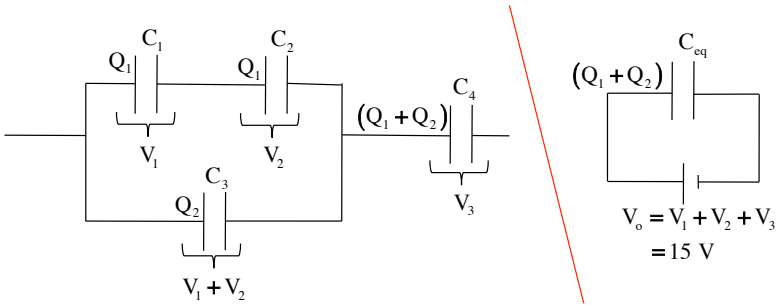


So with adding in the power of ten:

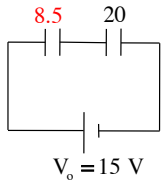
$$C_{equ}=\frac{C_{8.5}C_{20}}{C_{8.5}+C_{20}}=\frac{8.5(20)}{8.5+20}=5.96\text{x}10^{-6}\ \text{F}$$

b.) What is the charge on each capacitor?

It isn't a bad idea to think a little about the quantities that are common to the various components within the system. The circuit below lists the voltage, capacitance and charge for each of the elements of the original circuit and for the equivalent circuit.



The equivalent capacitance is:



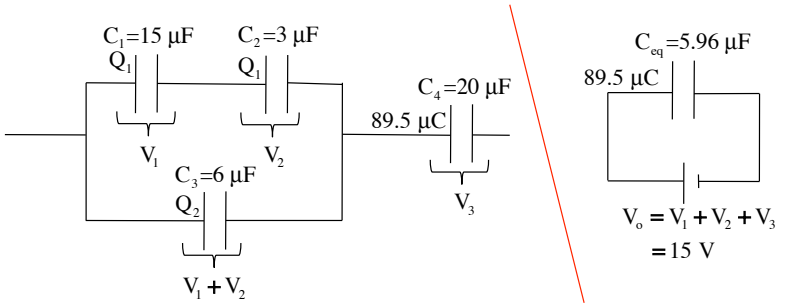
$$C_{eq}=\frac{Q}{V}$$

$$\Rightarrow\quad Q=C_{eq}V_o$$

$$=(5.96\text{x}10^{-6}\ \text{F})(15\ \text{V})$$

$$=89.5\text{x}10^{-6}\ \text{C}$$

Looking back at the previous page, we can see that the charge on C_{eq} is the same as the charge on the 20 μf cap, which is 89.5 μf, so we've taken care of that capacitor. As for the rest of the caps, we need to know their voltages. Filling in our circuit and playing around with what we know:



Doodling:

$$V_3=\frac{89.5\ \mu\text{C}}{20\ \mu\text{F}}$$

$$=4.47\ \text{V}$$

So

$$V_o=(15\ \text{V})=V_1+V_2+(4.47\ \text{V})$$

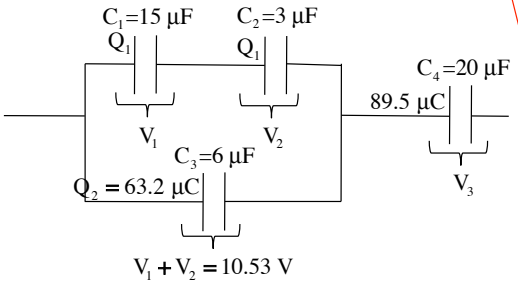
$$\Rightarrow\ V_1+V_2=10.53\ \text{V}$$

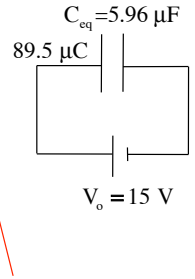
So

$$Q_2=C_3(V_1+V_2)$$

$$=(6.00\text{x}10^{-6}\ \text{F})(10.53\ \text{V})$$

$$=63.2\text{x}10^{-6}\ \text{F}$$





So if we were masochists, we could start with $V_1 + V_2 = 10.53\ \text{V}$, note that $V = Q/C$, and solve for the Q's . . . or we could be clever and note that if $(Q_1 + Q_2) = 89.5\ \mu\text{F}$, and if, as we've deduced, $Q_2 = 63.2\ \mu\text{C}$, then Q_1 must be $26.3\ \mu\text{C}$. QED (hee, hee).